DHANAMANJURI UNIVERSITY

DECEMBER-2025

Name of Programme : M.A./M.Sc. Mathematics
Semester : 1st
Paper Code : MAT-504
Paper Title : Complex Analysis-I
Full Marks : 80
Pass Marks : 32 **Duration: 3 hours**

The figures in the margin indicate full marks for the questions.

Answers all the questions:

UNIT-I

Answer any three from the following questions: **$10 \times 3 = 30$**

1. a) When is a complex function $f(z)$ said to have a limit w_0 at $z = z_0$? 1

   b) Prove that $\lim_{z \to 2+3i} (x + i(x + y^2)) = 2 + 11i$ using $\varepsilon - \delta$ definition. 4

   c) Let $f(z) = u(x,y) + iv(x,y), z_0 = x_0 + iy_0$ and $w_0 = p + iq$. Then prove that
   $\lim_{z \to z_0} f(z) = w_0 \Leftrightarrow \lim_{(x,y)\to(x_0,y_0)} u(x,y) = p \;\&\; \lim_{(x,y)\to(x_0,y_0)} v(x,y) = q.$ 5

2. a) Let $f(z)$ be a continuous function on a closed and bounded set S in a complex plane. Then prove that f is uniformly continuous on S. 5

   b) Show that $f(z) = z^2$ is uniformly continuous in the region $|z| < 1$ but $g(z) = \dfrac{1}{z}$ is not uniformly continuous in the same region. 5

3. a) Consider the function $f(z) = |z|$. Prove that it is nowhere differentiable in $\mathbb{C}$ but continuous everywhere. 3

b) For the function $f(z)=\begin{cases}\frac{\bar{z}^2}{z}, & z\neq 0\\ 0, & z=0\end{cases}$ show that the Cauchy-Riemann equations are satisfied at $(0,0)$ but the function is not differentiable at $z=0$. 7

4. Establish the complex form of Cauchy-Riemann equation and hence express $f'(z)$ in complex form. 10

5. a) Integrate the function $f(z)=x+y-3ix^3$ between the points $z=0$ and $z=1+i$ along two different paths. 2

b) Let $f(z)$ be analytic in a domain Ω and let γ be a simple closed contour in Ω, taken in positive sense. Then for all z interior to γ prove that $f'(z)=\frac{1}{2\pi i}\int_\gamma \frac{f(s)}{(s-z)^2}ds$. 8

UNIT-II

Answer any three from the following questions: **$10\times 3=30$**

6. If a function f is analytic in the annular domain $R_1<|z-a|<R_2$ and C be any positively oriented circle around a and lying in this domain then prove that f can be expanded about any point in the annular domain $R_1<|z-a|<R_2$ as $f(z)=\sum_{n=0}^{\infty}a_n(z-a)^n+\sum_{n=1}^{\infty}\frac{b_n}{(z-a)^n}$ where

$$a_n=\frac{1}{2\pi i}\int_C \frac{f(s)}{(s-a)^{n+1}}ds,\ n=0,1,2,\ldots \text{ and}$$

$$b_n=\frac{1}{2\pi i}\int_C \frac{f(s)}{(s-a)^{-n+1}}ds,\ n=1,2,\ldots$$ 10

7. Define the following with examples: 10
   a) Isolated Singularity,
   b) Non-Isolated Singularity,
   c) Removable Singularity,
   d) Pole,
   e) Essential Singularity

8. a) State and prove Cauchy Residue Theorem. 5

b) Find the singularities of the function $f(z)=\dfrac{z^2+16}{(z-i)^2(z+3)}$ and hence its type as well as the residue. 5

9. Obtain the Laurent Series expansion of $f(z)=\dfrac{z}{z^2-4z+3}$. Also expand it in powers of $z-1$ and mention the region of validity. 10

10. Find the singularities of the following functions and hence it's type and Residue:

a) $f(z)=\dfrac{e^{-z}}{(z-2)^4}$

b) $f(z)=\dfrac{\pi\cos\pi z}{z^2}$ 10

UNIT-III

Answer any two from the following questions: **$10\times 2=20$**

11. Define Conformal Mapping. Using the basic definition of Conformality, show that the mapping $w=f(z)=\dfrac{1}{z}$ is Conformal at $z=1$. 10

12. a) Define Invariant point of a bilinear transformation. Write the canonical form of a bilinear transformation having two invariant points. 2

b) Show that every bilinear transformation maps circles and lines into circles and lines.

13. Define Cross ratio of three distinct complex numbers. Find the bilinear transformation which maps $i, 1, -1$ onto $1, 0, \infty$. 10

14. Find the bilinear transformation that maps the crescent-shaped region that lies inside the circle $|z-2|<2$ and outside the circle $|z-1|>1$ onto a horizontal strip in the upper half plane. 10

*****